## What have we learned?

We've learned to work with random variables. We can use the probability model for a discrete random variable to find its expected value and its standard deviation.

We've learned that the mean of the sum or difference of two random variables, discrete or continuous, is just the sum or difference of their means. And we've learned the Pythagorean Theorem of Statistics: *For independent random variables,* the variance of their sum or difference is always the *sum* of their variances.

Finally, we've learned that Normal models are once again special. Sums or differences of Normally distributed random variables also follow Normal models.

## TERMS

**Random variable** A random variable assumes any of several different values as a result of some random event. Random variables are denoted by a capital letter such as $X$.

**Discrete random variable** A random variable that can take one of a finite number[3] of distinct outcomes is called a discrete random variable.

**Continuous random variable** A random variable that can take any numeric value within a range of values is called a continuous random variable. The range may be infinite or bounded at either or both ends.

**Probability model** The probability model is a function that associates a probability $P$ with each value of a discrete random variable $X$, denoted $P(X = x)$, or with any interval of values of a continuous random variable.

**Expected value** The expected value of a random variable is its theoretical long-run average value, the center of its model. Denoted $\mu$ or $E(X)$, it is found (if the random variable is discrete) by summing the products of variable values and probabilities:

$$\mu = E(X) = \sum x \cdot P(x).$$

**Variance** The variance of a random variable is the expected value of the squared deviation from the mean. For discrete random variables, it can be calculated as:

$$\sigma^2 = Var(X) = \sum (x - \mu)^2 P(x).$$

**Standard deviation** The standard deviation of a random variable describes the spread in the model, and is the square root of the variance:

$$\sigma = SD(X) = \sqrt{Var(X)}.$$

**Changing a random variable by a constant:**

$$E(X \pm c) = E(X) \pm c \qquad Var(X \pm c) = Var(X)$$
$$E(aX) = aE(X) \qquad Var(aX) = a^2 Var(X)$$

**Adding or subtracting random variables:** $E(X \pm Y) = E(X) \pm E(Y)$ and *if* $X$ and $Y$ *are independent,* $Var(X \pm Y) = Var(X) + Var(Y)$ (The Pythagorean Theorem of Statistics).

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[3] Technically, there could be an infinite number of outcomes as long as they're *countable.* Essentially that means we can imagine listing them all in order, like the counting numbers 1, 2, 3, 4, 5, . . .

## *SKILLS* *When you complete this lesson you should:*

**Think**

- Be able to recognize random variables.
- Understand that random variables must be independent in order to determine the variability of their sum or difference by adding variances.

**Show**

- Be able to find the probability model for a discrete random variable.
- Know how to find the mean (expected value) and the variance of a random variable.
- Always use the proper notation for these population parameters, $\mu$ or $E(X)$ for the mean, and $\sigma$, $SD(X)$, $\sigma^2$, or $Var(X)$ when discussing variability.
- Know how to determine the new mean and standard deviation after adding a constant, multiplying by a constant, or adding or subtracting two independent random variables.

**Tell**

- Be able to interpret the meaning of the expected value and standard deviation of a random variable in the proper context.

## Random Variables on the Computer

Statistics packages deal with data, not with random variables. Nevertheless, the calculations needed to find means and standard deviations of random variables are little more than weighted means. Most packages can manage that, but then they are just being overblown calculators. For technological assistance with these calculations, we recommend you pull out your calculator.

## EXERCISES

**1. Expected value.** Find the expected value of each random variable:

a)

| $x$ | 10 | 20 | 30 |
|---|---|---|---|
| $P(X = x)$ | 0.3 | 0.5 | 0.2 |

b)

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $P(X = x)$ | 0.3 | 0.4 | 0.2 | 0.1 |

**2. Expected value.** Find the expected value of each random variable:

a)

| $x$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X = x)$ | 0.2 | 0.4 | 0.4 |

b)

| $x$ | 100 | 200 | 300 | 400 |
|---|---|---|---|---|
| $P(X = x)$ | 0.1 | 0.2 | 0.5 | 0.2 |

**3. Pick a card, any card.** You draw a card from a deck. If you get a red card, you win nothing. If you get a spade, you win $5. For any club, you win $10 plus an extra $20 for the ace of clubs.

a) Create a probability model for the amount you win at this game.
b) Find the expected amount you'll win.
c) How much would you be willing to pay to play this game?

**4. You bet!** You roll a die. If it comes up a 6, you win $100. If not, you get to roll again. If you get a 6 the second time, you win $50. If not, you lose.

a) Create a probability model for the amount you win at this game.
b) Find the expected amount you'll win.
c) How much would you be willing to pay to play this game?

**5. Kids.** A couple plans to have children until they get a girl, but they agree that they will not have more than three children even if all are boys. (Assume boys and girls are equally likely.)
a) Create a probability model for the number of children they'll have.
b) Find the expected number of children.
c) Find the expected number of boys they'll have.

**6. Carnival.** A carnival game offers a $100 cash prize for anyone who can break a balloon by throwing a dart at it. It costs $5 to play, and you're willing to spend up to $20 trying to win. You estimate that you have about a 10% chance of hitting the balloon on any throw.
a) Create a probability model for this carnival game.
b) Find the expected number of darts you'll throw.
c) Find your expected winnings.

**7. Software.** A small software company bids on two contracts. It anticipates a profit of $50,000 if it gets the larger contract and a profit of $20,000 on the smaller contract. The company estimates there's a 30% chance it will get the larger contract and a 60% chance it will get the smaller contract. Assuming the contracts will be awarded independently, what's the expected profit?

**8. Racehorse.** A man buys a racehorse for $20,000 and enters it in two races. He plans to sell the horse afterward, hoping to make a profit. If the horse wins both races, its value will jump to $100,000. If it wins one of the races, it will be worth $50,000. If it loses both races, it will be worth only $10,000. The man believes there's a 20% chance that the horse will win the first race and a 30% chance it will win the second one. Assuming that the two races are independent events, find the man's expected profit.

**9. Variation 1.** Find the standard deviations of the random variables in Exercise 1.

**10. Variation 2.** Find the standard deviations of the random variables in Exercise 2.

**11. Pick another card.** Find the standard deviation of the amount you might win drawing a card in Exercise 3.

**12. The die.** Find the standard deviation of the amount you might win rolling a die in Exercise 4.

**13. Kids.** Find the standard deviation of the number of children the couple in Exercise 5 may have.

**14. Darts.** Find the standard deviation of your winnings throwing darts in Exercise 6.

**15. Repairs.** The probability model below describes the number of repair calls that an appliance repair shop may receive during an hour.

| **Repair Calls** | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| **Probability** | 0.1 | 0.3 | 0.4 | 0.2 |

a) How many calls should the shop expect per hour?
b) What is the standard deviation?

**16. Red lights.** A commuter must pass through five traffic lights on her way to work and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown below.

| **X = # of red** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **P(X = x)** | 0.05 | 0.25 | 0.35 | 0.15 | 0.15 | 0.05 |

a) How many red lights should she expect to hit each day?
b) What's the standard deviation?

**17. Defects.** A consumer organization inspecting new cars found that many had appearance defects (dents, scratches, paint chips, etc.). While none had more than three of these defects, 7% had three, 11% two, and 21% one defect. Find the expected number of appearance defects in a new car and the standard deviation.

**18. Insurance.** An insurance policy costs $100 and will pay policyholders $10,000 if they suffer a major injury (resulting in hospitalization) or $3000 if they suffer a minor injury (resulting in lost time from work). The company estimates that each year 1 in every 2000 policyholders may have a major injury, and 1 in 500 a minor injury.
a) Create a probability model for the profit on a policy.
b) What's the company's expected profit on this policy?
c) What's the standard deviation?

**19. Contest.** You play two games against the same opponent. The probability you win the first game is 0.4. If you win the first game, the probability you also win the second is 0.2. If you lose the first game, the probability that you win the second is 0.3.
a) Are the two games independent? Explain your answer.
b) What's the probability you lose both games?
c) What's the probability you win both games?
d) Let random variable $X$ be the number of games you win. Find the probability model for $X$.
e) What are the expected value and standard deviation of $X$?

**20. Contracts.** Your company bids for two contracts. You believe the probability you get contract #1 is 0.8. If you get contract #1, the probability you also get contract #2 will be 0.2, and if you do not get #1, the probability you get #2 will be 0.3.
a) Are the two contracts independent? Explain.
b) Find the probability you get both contracts.
c) Find the probability you get no contract.
d) Let $X$ be the number of contracts you get. Find the probability model for $X$.
e) Find the expected value and standard deviation of $X$.

**21. Batteries.** In a group of 10 batteries, 3 are dead. You choose 2 batteries at random.

a) Create a probability model for the number of good batteries you get.
b) What's the expected number of good ones you get?
c) What's the standard deviation?

**22. Kittens.** In a litter of seven kittens, three are female. You pick two kittens at random.
a) Create a probability model for the number of male kittens you get.
b) What's the expected number of males?
c) What's the standard deviation?

**23. Random variables.** Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:
a) $3X$
b) $Y + 6$
c) $X + Y$
d) $X - Y$
e) $X_1 + X_2$

| | Mean | SD |
|---|---|---|
| **X** | 10 | 2 |
| **Y** | 20 | 5 |

**24. Random variables.** Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:
a) $X - 20$
b) $0.5Y$
c) $X + Y$
d) $X - Y$
e) $Y_1 + Y_2$

| | Mean | SD |
|---|---|---|
| **X** | 80 | 12 |
| **Y** | 12 | 3 |

**25. Random variables.** Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:
a) $0.8Y$
b) $2X - 100$
c) $X + 2Y$
d) $3X - Y$
e) $Y_1 + Y_2$

| | Mean | SD |
|---|---|---|
| **X** | 120 | 12 |
| **Y** | 300 | 16 |

**26. Random variables.** Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:
a) $2Y + 20$
b) $3X$
c) $0.25X + Y$
d) $X - 5Y$
e) $X_1 + X_2 + X_3$

| | Mean | SD |
|---|---|---|
| **X** | 80 | 12 |
| **Y** | 12 | 3 |

**27. Eggs.** A grocery supplier believes that in a dozen eggs, the mean number of broken ones is 0.6 with a standard deviation of 0.5 eggs. You buy 3 dozen eggs without checking them.
a) How many broken eggs do you expect to get?
b) What's the standard deviation?
c) What assumptions did you have to make about the eggs in order to answer this question?

**28. Garden.** A company selling vegetable seeds in packets of 20 estimates that the mean number of seeds that will actually grow is 18, with a standard deviation of 1.2 seeds. You buy 5 different seed packets.
a) How many bad seeds do you expect to get?
b) What's the standard deviation?
c) What assumptions did you make about the seeds? Do you think that assumption is warranted? Explain.

**29. Repair calls.** Find the mean and standard deviation of the number of repair calls the appliance shop in Exercise 15 should expect during an 8-hour day.

**30. Stop!** Find the mean and standard deviation of the number of red lights the commuter in Exercise 16 should expect to hit on her way to work during a 5-day work week.

**31. Fire!** An insurance company estimates that it should make an annual profit of \$150 on each homeowner's policy written, with a standard deviation of \$6000.
a) Why is the standard deviation so large?
b) If it writes only two of these policies, what are the mean and standard deviation of the annual profit?
c) If it writes 10,000 of these policies, what are the mean and standard deviation of the annual profit?
d) Do you think the company is likely to be profitable? Explain.
e) What assumptions underlie your analysis? Can you think of circumstances under which those assumptions might be violated? Explain.

**32. Casino.** A casino knows that people play the slot machines in hopes of hitting the jackpot, but that most of them lose their dollar. Suppose a certain machine pays out an average of \$0.92, with a standard deviation of \$120.
a) Why is the standard deviation so large?
b) If you play 5 times, what are the mean and standard deviation of the casino's profit?
c) If gamblers play this machine 1000 times in a day, what are the mean and standard deviation of the casino's profit?
d) Do you think the casino is likely to be profitable? Explain.

**33. Cereal.** The amount of cereal that can be poured into a small bowl varies with a mean of 1.5 ounces and a standard deviation of 0.3 ounces. A large bowl holds a mean of 2.5 ounces with a standard deviation of 0.4 ounces. You open a new box of cereal and pour one large and one small bowl.
a) How much more cereal do you expect to be in the large bowl?
b) What's the standard deviation of this difference?
c) If the difference follows a Normal model, what's the probability the small bowl contains more cereal than the large one?
d) What are the mean and standard deviation of the total amount of cereal in the two bowls?
e) If the total follows a Normal model, what's the probability you poured out more than 4.5 ounces of cereal in the two bowls together?